Minimal conductivity of rippled graphene with topological disorder

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We study the transport properties of a neutral graphene sheet with curved regions induced or stabilized by topological defects. The proposed model gives rise to Dirac fermions in a random magnetic field and in a random scalar potential acting like a space-dependent Fermi velocity induced by the curvature. The last term leads to a singular long-range correlated disorder with special characteristics. The Drude minimal conductivity at zero energy is found to be inversely proportional to the density of topological disorder, a signature of diffusive behavior.

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I. INTRODUCTION

Since its experimental realization,[1–3] graphene has been a focus of intense research activity both theoretically and experimentally. The origin of this interest lies partially in the experimental capability of exploring the transport properties that show a set of interesting features related to disorder and which open the way to graphene-based electronics.[4]

One of the most intriguing properties of graphene is the observation of a minimal conductivity at zero frequency in undoped suspended samples that in early measurements was argued to have a universal value of the order of e^2/h (Refs. 2, 5, and 6) independent of the disorder concentration and a factor of π bigger than the one predicted by theory.[7–17] Other experiments in both mechanically deposited graphene and graphene grown on a substrate[3,18,19] have found a bigger dispersion in the coefficient of the universal behavior while more recent calculations[20–22] have casted some doubts on the disorder dependence of the numerical coefficient and the actual situation remains unclear.

Another peculiarity of most of the graphene samples is the existence of mesoscopic corrugations[19,23,24] whose possible influence on transport properties only now starts to be explored.[23,25–28] Although the observed ripples were invoked from the very beginning to explain the absence of weak localization in the samples,[6,29,30] there have been so far few attempts to model the corrugations based on either the curved space approach with[31,32] or without topological defects,[33] or the theory of elasticity.[23,34–36] The possible physical implications of the ripples in connection with the charge anisotropies has been revisited in very recent works.[37,38]

In Refs. 31 and 32, we proposed a model for rippled graphene based on the presence of defective rings (pentagons and heptagons) in the samples. These types of topological defects were observed before in nanotubes and in bombarded graphite and are known to be a natural way to get rid of tensions in the hexagonal lattice.[39] They have also very recently been produced and observed with transmission electron microscopy in suspended graphene samples.[40] It is also natural to think that some of the defects that were either present in the graphite sample or formed during the very energetic procedure of mechanical cleavage, stay quenched in the two-dimensional samples.

It is clear by now that the low-energy electronic properties of graphene are very well described by the massless Dirac equation in two dimensions, a fact coming from the symmetries of the hexagonal lattice, and also obtained in the tight-binding approximation.[41] It is also known that the Dirac points are very robust to deformations of the lattice[42] so we will model curved graphene assuming that the Dirac points are not affected by the presence of ripples and hence that the curved samples can be described by writing the Dirac equation in the given curved surface.[43] Within this formalism, we studied recently the electronic structure of the sample with topological defects[31,32] and that of graphene with smooth curved regions.[33] In the last work, we emphasized the fact that curvature gives rise not only to an effective magnetic field, a property known from the early times of graphene,[44–46] but also to an effective position-dependent Fermi velocity which can have strong influence on the physical properties of the system.

In this work, we continue studying the physical properties of curved graphene within the geometrical approach. The electronic properties of the system were explored in Refs. 31 and 32 by computing the two-point Green's function of the electron. There, we were able to make advances keeping a fixed number of defects at given positions of the lattice. Studying the transport properties is a much more difficult task. We need to assume a density of defects with some statistical distribution and to average over defects. We apply the standard techniques of disordered electrons[47] to rippled graphene by averaging over the random effective potential induced by curvature. We will make special emphasis on the case of having topological defects. The smooth curvature case can be implemented easily. We find that averaging over the different metric factors in the case of having a nontrivial metric induced by topological defects gives rise to a long-range correlated interaction that affects severely the one-particle properties of the system. We compute semiclassically the zero frequency conductivity and find that it depends on the inverse of the density of disorder, a behavior characteristic of diffusive systems. We argue that even if the Drude value obtained in this work is renormalized by quantum corrections to the universal minimal conductivity, there will still be a region in parameter space where this model differs from the ones studied previously.

This paper is organized as follows. In Sec. II we review the model of topological defects and establish the effective

Hamiltonian. In Sec. III, we define the statistical properties of the fields induced by the defects and apply the replica trick to get the effective four Fermi interaction. We see that the interaction is anisotropic and singular in the forward direction, a behavior due to the long-range character of the conical singularities. We discuss this issue and extract the interaction coefficients that will appear in the computation of the lifetime and the density of states of the disordered system. Section IV contains the semiclassical expansion of the sigma model to get the low-energy behavior of the system. We discuss two possible saddle points and compute the effective potential to establish the symmetry-breaking minimum, a basic point to the rest of the calculation. From there, we deduce, after some calculations detailed in the Appendix, the diffusive behavior of the system. Section V contains a discussion of the results and open questions.

II. MODEL

In this section, we follow closely Refs. 31 and 32 to describe the explicit form of the potential generated by the defects and its statistical properties. The model for corrugated graphene is based on the presence of defective carbon rings at arbitrary positions in the lattice. It is known that the substitution of an hexagon by an n-sided polygon with n greater (smaller) than six gives rise to locally curved portions in the sample with positive (negative) curvature. It was argued that the presence of an equal number of heptagons and pentagons would keep the sample flat in average and mimic the corrugations observed in the free-standing samples. In order to study transport properties, we need to consider a density of defects located at random positions.

The behavior of the electrons in curved graphene is described by the Hamiltonian

$$H = iv_F\int d^2\mathbf{r}\sqrt{g}\bar{\psi}\gamma^{\mu}[\partial_{\mu} - \Omega_{\mu}(\mathbf{r})]\psi. \tag{1}$$

The curved Dirac matrices $\gamma^{\mu}(\mathbf{r})$ are related with the usual constant matrices γ^{a} by

$$\gamma^{\mu}(\mathbf{r}) = e^{\mu}_{a}(\mathbf{r})\gamma^{a},$$

where e^{μ}_{a} is the tetrad constructed with the metric tensor. It is this factor in combination with the determinant of the metric $\sqrt{g}$ that gives rise to a long-range correlated scalar field. In a tight-binding scheme, this term can be modeled as a global modulation of the nearest neighbor hopping[37] or of the average distance between carbon atoms[38] induced by curvature. The term Ω_{μ} contains the spin connection and the possible extra gauge fields induced by the defects.[39] It is given by

$$\Omega_{\mu}(\mathbf{r}) = \Gamma_{\mu}(\mathbf{r}) - \tau_{\alpha}A^{\alpha}_{\mu}(\mathbf{r}), \tag{2}$$

where Γ_{μ} is due to the spin connection and the nonabelian part $\tau_{\alpha}A^{\alpha}_{\mu}$ is related to the holonomy and will be discussed later.

In Refs. 32, we described the curved space generated by an arbitrary number of topological defects located at positions $\mathbf{r}_j$ by the metric

$$g_{ij} = e^{\Lambda(\mathbf{r})}\delta_{ij}, \tag{3}$$

where the conformal factor $\Lambda(\mathbf{r})$ takes the form

$$\Lambda(\mathbf{r}) = \sum_{j=1}^{N}\frac{\mu_j}{2\pi}\log\left|\frac{\mathbf{r}-\mathbf{r_j}}{\mathbf{a}^*}\right|, \tag{4}$$

where μ_j is a constant related to the defect (or excess) angle of the disclinations and a^* is a constant of the order of the lattice spacing, interpreted as the radius of the "core" of the defect. The specific form of the conformal factor (4) gave rise to strongly diverging one-particle properties such as the local density of states. We will see that in the process of averaging over disorder one-particle properties will remain singular while two-particle properties, such as the Drude conductivity, will be finite.

It is well known that the presence of topological defects has other consequences besides that of the inducing local curvature to the graphene sheet. Odd membered rings mix the two Fermi points what can be modeled by nonabelian gauge fields.[44,45] Also if various defects are present, an extra phase appears due to the noncommutativity of the holonomy operators associated to the valley mixing phase and the proper Berry phase acquired by the fermions when surrounding the defects.[48,49] All these phases are naturally incorporated in the formalism by generic external nonabelian gauge fields $A_{\alpha}(\mathbf{r})$ in Eq. (1). This also accounts for the classification of the various disorder types described in Refs. 14 and 50. In the case of having an equal number of pentagonal and heptagonal defects, it can be shown that only the (abelian) gauge field associated to the conical singularity remains[39] and we can restrict ourselves to the scattering problem around a single Fermi point. We will comment on the possible modifications of the calculation that a more general case would induce in Sec. V.

The value of $|\mu_j| \equiv \mu$ is $\frac{1}{24}$ for both pentagon and heptagon rings that differ in its sign. We use μ as a perturbative parameter around flat space and expand the determinant of the metric $g(\mathbf{r})$, the zweibeins $e^{\mu}_{a}(\mathbf{r})$, and the spin connection $\Omega(\mathbf{r})$ in Eq. (1). To first order in μ the Hamiltonian is

$$H = iv_F\int d^2\mathbf{r}\left[(1+\Lambda(\mathbf{r}))\bar{\psi}\gamma^{j}\partial_i\psi + \frac{i}{2}\bar{\psi}\gamma^{j}(\partial_i\Lambda(\mathbf{r}))\psi\right]. \tag{5}$$

Equations (4) and (5) are the basis of our calculation. We have a term associated to the conformal factor $\Lambda(\mathbf{r})$ that can be described in flat space as a disorder-induced modification of the Fermi velocity, and a gauge-field-type term if we identify $A_i \sim \partial_i\Lambda(\mathbf{r})$. Notice that although the gauge potential A_i seems to be a total derivative, the effective magnetic field is not zero due to the singular form of the function $\Lambda(\mathbf{r})$ [Eq. (4)].

III. AVERAGING OVER DISORDER

We will study the transport properties of this topologically disordered graphene by assuming a random distribution of an equal number of pentagons and heptagons. The statistical properties of these topological defects were analyzed in part in Ref. 51. It was proposed there that the Gaussian disorder induced by a random distribution of topological defects can be described by a single dimensionless quantity Δ proportional to the average fluctuations of the non-abelian-vector potential representing a vortex at the position of the defect,

$$\langle \mathbf{A}(\mathbf{r})\mathbf{A}(\mathbf{r}')\rangle = \Delta \delta^2(\mathbf{r}-\mathbf{r}'). \tag{6}$$

The infrared behavior of Δ was analyzed in Ref. 51 and shown to diverge logarithmically with the size R of the system in the case of unbounded disclinations,

$$\Delta = 2\pi\Phi_0^2 \log\left(\frac{R}{l_0}\right), \tag{7}$$

while it remains of constant value,

$$\Delta \propto \Phi_0^2 n_{\text{disl}} b^2, \tag{8}$$

in the case of having pentagon-heptagon pairs bounded into dislocations with density n_{disl} and with an average distance b.

We will consider a case in which we have a random distribution of an equal number of five and seven rings not bounded into dislocations but at average distances d such that the "volume" occupied by virtual strings pairing the defects is small compared with the total size of the sample $(d/L)^2 \ll 1$. This allows us to neglect the gauge fields associated to the Z_3 electronic holonomy described in Refs. 48 and 49. This assumption is reasonable if the total density of defects is small as should be the case in the experimental samples that exhibit all the properties of the clean honeycomb lattice.

The most important issue in this work is the new random field $\Lambda(\mathbf{r})$ associated to the Fermi velocity modification given in Eq. (4). We will assume for this scalar field a zero mean value $\langle\Lambda(\mathbf{r})\rangle=0$ and

$$\langle \Lambda(\mathbf{r})\Lambda(\mathbf{r}')\rangle = na^{*2} \log\left|\frac{\mathbf{r}-\mathbf{r}'}{a^*}\right|, \tag{9}$$

where n is proportional to the areal density of defects and a^* is of the order of the lattice spacing. This correlator diverges in both the infrared and the ultraviolet limits. It induces over the random magnetic field $A_i(\mathbf{r})$ the average

$$\langle A_i(\mathbf{r})A_j(\mathbf{r}')\rangle \sim na^{*2}\delta_{ij}\delta(\mathbf{r}-\mathbf{r}'). \tag{10}$$

The special form of the correlator [Eq. (9)] can be understood when we consider the nature of the defects. In a geometrical description of defects in two-dimensional crystals,[52] the equations of motion for the metric tensor $g_{ij}(\mathbf{r})$ reduce to a unique equation for the conformal factor [Eq. (4)]

$$\nabla^2\Lambda(\mathbf{r}) = \sum_j^N \frac{\mu_j}{2\pi}\delta(\mathbf{r}-\mathbf{r}_j). \tag{11}$$

We can rewrite Eq. (4) in terms of the Green's function $K(\mathbf{r}-\mathbf{r}')$ of Eq. (11),

$$\Lambda(\mathbf{r}) = \sum_{j=1}^N \frac{\mu_j}{2\pi}\int d\mathbf{r}'\,\delta(\mathbf{r}'-\mathbf{r}_j)K(\mathbf{r}-\mathbf{r}'), \tag{12}$$

where the asymptotic behavior of $K(\mathbf{r}-\mathbf{r}')$ for distances $\mathbf{r} \gg a^*$ is

$$K(\mathbf{r}-\mathbf{r}') \approx \log\left|\frac{\mathbf{r}-\mathbf{r}'}{a^*}\right|, \tag{13}$$

what justifies Eq. (9).

In momentum space the correlators are

$$\langle \Lambda(\mathbf{p})\Lambda(-\mathbf{p})\rangle = \frac{na^{*2}}{p^2}, \tag{14}$$

$$\langle A_i(\mathbf{p})A_j(-\mathbf{p})\rangle = na^{*2}\delta_{ij}. \tag{15}$$

Notice that the Λ term describes a new type of disorder; Dirac Fermions in two space dimensions with a random velocity, a problem that, to our knowledge, has not been addressed in the early literature.[53–56]

In momentum space representation the action corresponding to Eq. (5) reads

$$\begin{aligned} S = \frac{1}{2}\int d\omega\ dk\ \bar{\psi}(\omega,\vec{k})(\omega\gamma^0 - v_F\vec{\gamma}\vec{k})\psi(\omega,\vec{k}) \\ - \frac{v_F}{2}\int dp\ dk\Lambda(p)\bar{\psi}(\omega,\vec{k})\vec{\gamma}\vec{k}\ \psi(\omega,\vec{k}) \\ - \frac{i}{2}v_F\int dk\frac{1}{2}\bar{\psi}(\omega,\vec{k})\vec{\gamma}\vec{A}(\vec{k})\psi(\omega,\vec{k}), \end{aligned} \tag{16}$$

where $dk \equiv \frac{d^2\mathbf{k}}{4\pi^2}$ and

$$\Lambda(\mathbf{p}) = \int \frac{d^2x}{2\pi}e^{ipx}\log\left|\frac{x}{a}\right|. \tag{17}$$

The double integral in the middle term of Eq. (16) comes from the Fourier transform of the product $\Lambda(x)\partial_x$ which results in a convolution

$$\Lambda(x)\partial_x \to \int \hat{\Lambda}(s)(k-s)ds.$$

After reshifting the momentum, we get Eq. (16). We integrate out Λ using Eq. (14) as a quadratic action. It is interesting to note that this term is similar to the interaction between curvatures in a continuous hexatic membrane where $1/na^{*2}$ plays the role of the hexatic stiffness constant.[52]

Replicating the fields and integrating out Λ in Eq. (16) we get

$$S = S_0 + v_F^2\frac{\lambda}{2}\int dk\ dk'\Gamma(k,k')(\bar{\psi}_a\gamma^j\psi_a)(\bar{\psi}_b\gamma_i\psi_b), \tag{18}$$

where summation over replica indices a,b is assumed, $\lambda = 2\pi\mu^2 na^2$ is a dimensionless parameter proportional to the density of defects n and

$$\Gamma(k,k') = \left(\frac{(k+k')^2}{(k-k')^2} - \frac{1}{4}\right). \tag{19}$$

The constant term in the interaction vertex [Eq. (19)] comes from the random magnetic field and would give rise to the standard result found in previous works. The term coming from the Fermi velocity is anisotropic and singular when $k \to k'$, a signature of the infrared singularity associated to the effect of a conical defect at infinite distances from the apex. Because we are dealing with elastic scattering, the modulus of the momentum is conserved provided that the energy ω in the process is conserved. The function $\Gamma(k,k')$ in Eq. (19) can be written as a function of the difference of the scattering angles, $\phi \equiv \theta_k - \theta_{k'}$,

$$\Gamma(\phi) = \left(\frac{\cos^2(\phi/2)}{\sin^2(\phi/2)} - \frac{1}{4}\right). \tag{20}$$

This function diverges at scattering angles $\phi=0$ (forward scattering) and the one-particle properties of the system such as the density of states will show anomalous behavior when compared with their counterparts in short-ranged scattering processes. The divergence in Eq. (20) can be regularized by including a cutoff δ,

$$\Gamma(\phi,\delta) = \left(\frac{\cos^2(\phi/2)}{\sin^2(\phi/2)+\delta^2} - \frac{1}{4}\right). \tag{21}$$

The meaning of the cutoff can be understood from the origin of the correlator for the function $\Lambda(\mathbf{r})$ in Eq. (9). Instead of having a long-ranged propagator corresponding to infinite range defects that behave like $\frac{1}{p^2}$, we may consider a correlator of the type

$$K(\mathbf{p}) \sim \frac{1}{p^2+\delta^2}, \tag{22}$$

which corresponds in real space to changing Eq. (13) by a modified Bessel function of the second kind,

$$K(\mathbf{r}-\mathbf{r}') = \mathbf{K_0}(\delta|\mathbf{r}-\mathbf{r}'|), \tag{23}$$

whose leading term in a small δ expansion is

$$K(\mathbf{r}-\mathbf{r}') \approx \log(\delta|\mathbf{r}-\mathbf{r}'|). \tag{24}$$

Using Eq. (22) instead of Eq. (14), we arrive at Eq. (21), where we have redefined δ as $\delta = \delta/k^2$. We can assume that the momenta involved in the problem are of the order of the wavelength λ of the states near the Dirac points (or the localization length if those states are localized) so we can consider that δ is of the order of λ/χ where χ is the biggest length scale of the problem comparable to the system size L. We will make the important assumption that $\chi > \lambda$. We will see that the Drude conductivity is independent of δ and then well defined.

The free action in Eq. (18) is the usual,

$$S_0 = \int dk \bar{\psi}_a(\omega\gamma^0 - v_F\gamma k + i\eta M)\psi_a. \tag{25}$$

The replica indices run from 1 to $2N$. The first N indices are associated to advanced fields ($+i\eta$ in the free action), and the second ones to retarded fields, related to $-i\eta$, with the obvious definition of the matrix M,

$$M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}_{2N}. \tag{26}$$

This η term breaks explicitly the $O(2N)$ symmetry of the action [Eq. (18)] leading to possible massless excitations.[57]

In addition to the forward divergence, the scattering function [Eq. (21)] is highly anisotropic[58] which will lead us to deal with a multichannel scattering problem in contrast to the sort ranged scattering case where the scattering only occurs in the s channel. We will see that despite the anisotropic scattering the diffusion process will have an isotropic behavior described by a scalar diffusion constant.

In order to keep track of the role of each channel, we decompose $\Gamma(\phi)$ in harmonics,

$$\Gamma(\phi) = \sum_n \Gamma_n e^{-in\phi}. \tag{27}$$

We will see later that only the $n=0, \pm 1$ channels will play a role (notice that $\Gamma_n = \Gamma_{-n}$). Their explicit values as a function of the cutoff δ are

$$\Gamma_0 = \frac{1}{\delta} - 5/4 + O(\delta), \quad \Gamma_1 = \frac{1}{\delta} - 2 + O(\delta). \tag{28}$$

IV. SADDLE POINT APPROXIMATION IN THE NONLINEAR σ MODEL AND RESULTS

In this section, we will construct the low-energy field theory describing the large scale behavior of the system (i.e., at length scales larger than the elastic mean-free path, l) following standard procedures.[47] The interaction term in Eq. (18) can be written as

$$S_{\text{int}} = \frac{1}{2}\lambda v_F^2 \sum_n \int (dk)(dk')\Gamma_n \chi_n(\hat{k})\chi_n^*(\hat{k}')(\bar{\psi}_a\gamma^i\psi_a)(\bar{\psi}_b\gamma_i\psi_b), \tag{29}$$

where $\chi_n(\hat{k}) = e^{in\theta_k}$. Now we proceed to simplify the model [Eq. (29)]. The current-current interaction in Eq. (29) can be transformed into a density-density-type term,[59]

$$S_{\text{int}} = -\frac{1}{2} g v_F^2 \sum_n \int dk dk' \Gamma_n \chi_n(\hat{k})\chi_n^*(\hat{k}') \times [(\bar{\psi}_a\psi_b)(\bar{\psi}_b\psi_a) - (\bar{\psi}_a\gamma_5\psi_b)(\bar{\psi}_b\gamma_5\psi_a)], \tag{30}$$

where we have redefined the coupling constant as $g \equiv -2\lambda$.

The four fermi interaction in Eq. (30) can be decoupled by means of a Hubbard-Stratonovitch transformation,

$$S = S_0 + \int dk \frac{1}{4\pi g v_F^2}[\sum_n \Gamma_n(Q_n^2 + \Pi_n^2) + i\sum_n \Gamma_n \chi_n(\hat{k})\bar{\psi}(Q_n - i\gamma_5\Pi_n)\psi], \tag{31}$$

where Q_n and Π_n are the Hubbard-Stratonovitch fields which are $2N$-dimensional matrix fields.

We can further simplify the calculations by performing a unitary transformation that diagonalizes the fermionic part of the action in Eq. (31), and leaves unchanged the functional integration measure,

$$\psi \to U\psi, \quad \bar{\psi} \to \hat{\psi}U^+, \quad Q_n \to UQ_nU^+, \quad \Pi_n \to U\Pi_nU^+, \quad \gamma_5 \to U\gamma_5U^+.$$

Without loss of generality we will name the transformed fields as the original ones. Integrating out the fermionic modes leads to the usual final form for the nonlinear σ model

$$S_{\text{eff}} = \int \mathrm{Tr}\ln G^{-1} - \frac{1}{4gv_F^2}\sum_n \Gamma_n(Q_n^2 + \Pi_n^2), \tag{32}$$

where the Green's function is defined by

$$G^{-1} = G_0^{-1} + i\eta M + i\sum_n \Gamma_n \chi_n(\hat{k})(Q_n - i\gamma_5\Pi_n) \tag{33}$$

and

$$G_0^{-1} = \begin{pmatrix} w + v_F k & 0 \\ 0 & w - v_F k \end{pmatrix} \otimes 1_{2N}. \tag{34}$$

Next we will make a saddle point approximation and seek for a solution of the saddle point equation

$$\frac{\delta S_{\text{eff}}}{\delta\langle Q_n\rangle} = 0. \tag{35}$$

In contrast to the case of isotropic short ranged scatterers[7,60] where a single equation is obtained, Eq. (35) represents an infinite number of coupled equations,

$$\frac{1}{2gv_F^2}\langle Q_n\rangle = \int (dk)\frac{\chi_n(\hat{k})}{G_0^{-1} + i\sum_n \Gamma_n\chi_n(\hat{k})\langle Q_n\rangle}. \tag{36}$$

In the limit $\omega \to 0$ and despite the fact that the scattering mechanism is anisotropic, we can try to find a solution of the type

$$\langle Q_n\rangle = f\delta_{n0}M. \tag{37}$$

Of course, the solution of the mean-field equations is not unique and other vacua with different properties may be found. The value of the trial function f is

$$f = \frac{v_F K}{\Gamma_0}(e^{2\pi/g\Gamma_0} - 1)^{-1/2}, \tag{38}$$

where K is an ultraviolet cutoff. Now we are free to make a choice over the mean field values of Q and Π. The standard choice is $\langle Q_0\rangle = \langle S_0\rangle$ and $\langle \Pi_0\rangle = 0$. With this solution for $\langle Q\rangle$ we get the lifetime

$$\frac{1}{2\tau} \equiv v_F K(e^{2\pi/g\Gamma_0} - 1)^{-1/2}, \tag{39}$$

which turns out to be a constant whose dependence on the disorder strength is typically nonperturbative $\tau^{-1}(g) \sim \exp(1/g)$. This value for the lifetime depends on both the ultraviolet cutoff K, and on the infrared cutoff δ through the scattering coefficient $\Gamma_0 \sim \frac{1}{\delta} - 5/4$. The ultraviolet cutoff can be removed by renormalization-group techniques[60] but the infrared cutoff δ remains and the one-particle properties of the theory depend explicitly on it.[61] We will see later that the transport properties become independent of the infrared cutoff.[62]

We can now compute the averaged density of states at the Fermi level,

$$\rho(0) = -\frac{1}{N\pi}\mathrm{Im}\int \frac{d^2k}{4\pi^2} G^R(k), \tag{40}$$

the index N in the denominator is the replica index that will be taken to zero at the end of the calculation. From now on we drop any reference to this limit.

The density of states as a function of the lifetime Eq. (39) is

$$\rho(0) = \frac{1}{g\Gamma_0 v_F^2}\frac{1}{2\tau}. \tag{41}$$

Before proceeding to compute the quantum fluctuations around the chiral symmetry-breaking solution of the saddle point equations, we will make a comment on the solution $\langle S_n\rangle = 0$. This solution leads to a vanishing density of states at the Fermi energy and to a sublinear frequency dependence $\rho(\omega) \sim |\omega|^\alpha$, with α being a function of the strength of the disorder, a behavior reported in.[53] In order to determine the true minimum of the Q field action, we have computed the effective potential as a function of the trial function f defined as in Eq. (38). This effective potential is the same as that of the Nambu-Jona Lasinio model[63–65] for $\langle \Pi_0\rangle = 0$,

$$V_{\text{eff}} = \frac{1}{4gv_F^2}f^2 + \frac{f^2}{4\pi}\left[\log\left(\frac{f^2}{K^2}\right) - 1\right]. \tag{42}$$

The result is well known. This potential has two critical solutions: $f = 0$ corresponding to a zero value for the effective potential and a symmetry-breaking solution

$$f = Ke^{1/2 - K^2/2}e^{-\pi/2g},$$

for which the value of the effective potential is $V_{\text{eff}} = -\frac{K^2}{4\pi}f^2$. This broken-symmetry solution equivalent to Eq. (38) is then a minimum of the theory.

We will then proceed computing the physical properties of the quantum field model built around the broken-symmetry solution. We will see that the physics obtained for this case is typically nonperturbative irrespective of the strength of the disorder.

The technical details of the rest of the computation are given in the Appendix. Once we have calculated the value of the leading configuration of Q_n from the saddle point Eqs. (35), we expand the action [Eq. (32)] around this value, setting $Q_n = \langle Q_n\rangle + \delta Q_n$ and retain in the expansion terms up to second order in δQ_n,

$$S \approx \langle S\rangle + \delta Q_n \frac{\delta^2 S^*}{\delta Q_n \delta Q_m}\delta Q_m + \cdots. \tag{43}$$

The * means that the functional derivative is evaluated at the saddle point solution for Q_n and Π_n.

The ultimate goal is to compute the action for the massless modes

$$\delta S = \int dq\, \delta Q_0 \frac{(2\tau)\Gamma_0}{4gv_F^2}(\eta + Dq^2)\delta Q_0, \tag{44}$$

from where we can extract the diffusion constant is D. From Eq. (44), we get

$$D = \frac{1}{2\pi^2}\frac{1}{g\rho(0)}\frac{1}{\Gamma_0 - \Gamma_1}, \tag{45}$$

where the coefficients Γ_0 and Γ_1 are given in Eq. (28) and their difference is $\Gamma_0 - \Gamma_1 = \frac{3}{4} + O(\delta)$, hence the diffusion coefficient is well defined when the cutoff δ is send to zero.

Finally, we can compute the semiclassical value for the static dc conductivity using the Einstein relation for the two diffusive channels δQ_0 and $\delta\Pi_0$ in Eq. (30)

$$\sigma_{\rm dc} = 4\frac{e^2}{\hbar}\rho(0)D = \frac{4e^2}{h}2\frac{4}{3\pi g}. \tag{46}$$

The factor of 4 comes from the spin and valley degeneracy and the factor 2 comes from the two diffusive channels. Because of the static nature of the disorder potential, the conductivity can acquire an extra factor of π coming from the chiral anomaly. This factor is independent of the disorder and would modify so that the complete conductivity is

$$\sigma_{\rm dc} \sim \frac{4e^2}{h}\frac{1}{\pi + \tilde{g}}, \tag{47}$$

where $\tilde{g}$ is proportional to $g = 4\pi\mu^2 na^2$, which contains information about the type of disorder (μ) and the density of disorder n.

V. DISCUSSION AND OPEN QUESTIONS

In this work, we have addressed the effects of curvature on the transport properties of corrugated graphene sheets. We have shown that coupling the Dirac field to a curved space gives rise to an effective potential whose general form and statistical properties depend on the metric. The main feature of the geometrical description is the appearance of an effective long-range correlated random scalar field coupled to the kinetic energy term in the Hamiltonian.

Smooth curved regions in graphene give rise to standard short-range correlated disorder as the one studied in the literature.[7] The presence of topological defects in the sample either as the main source of curvature or as a way to stabilize the ripples in the mechanically deposited samples gives rise to singular, long-range correlated disorder. The static conductivity of the system at the neutrality point implies diffusive behavior. A similar nonuniversal behavior has been found in the same system[21,22,66,67] and has been attributed to random coulomb scatterers present in the substrate. A crucial difference with the present work is that in the mentioned references the graphene sample is either heavily doped or it has a nonzero carrier density due to a local-field effect induced by the Coulomb impurities. In our work, the density of states is generated by the disorder as in Refs. 7 and 10.

Another noticeable feature of the model presented in this work is the strong dependence of the one-particle properties on the parameter δ regulating the infrared behavior of the model. The situation here is even worse than that of a two-dimensional electron gas in a long-range correlated random magnetic field discussed in Ref. 68. There, the one-particle relaxation time was found to diverge as the infrared cutoff is sent to zero but the finite density of states made the transport relaxation time τ_{tr} finite. In our case, τ_{tr} also depends on the density of states at the Fermi level but now the DOS is divergent for $\delta \to 0$. We have obtained a finite Drude conductivity due to the particular dependence of the diffusive constant with the density of states [Eq. (45)].

In Sec. III, we have introduced the parameter δ defined in a phenomenological fashion as the ratio between the characteristic length scale of the defect χ, and the wave or localization length λ of the states around the Fermi energy. In a semiclassical approximation to the problem, this parameter is essentially uncontrollable. We can nevertheless made an estimation of the range of applicability of our results by assuming that the localization length can be obtained from an analysis of the quantum corrections to the conductivity. The diffusive regime is characterized by a static mean-free path $l = v_F\tau_{tr}$ greater than the localization or wavelength λ but smaller than the system's size. The mean-free path can be estimated using expressions in Eqs. (39), (41), and (45) and assuming that $a^* \sim a$ and $\chi \sim L$. We thus find

$$l \sim \frac{2}{3\mu}\left(\frac{\chi}{\lambda}\right)^{1/2}\frac{1}{n_{imp}^{1/2}}, \tag{48}$$

from where we can get a lower bound for the density of defects in the case $\lambda < l$,

$$n_{imp} < \frac{4}{9\mu^2}\frac{L}{\lambda^3}. \tag{49}$$

In the same spirit, an upper bound can be estimated using the condition $l < L$,

$$n_{imp} > \frac{4}{9\mu^2}\frac{1}{L\lambda}. \tag{50}$$

In Refs. 13 and 69, the possible fixed points of the total conductivity where classified according to the symmetries of the original Hamiltonian in a renormalization-group analysis. As our disorder term preserves both chiral and time-reversal symmetries, the final conductivity once quantum corrections are taken into account should flow to the universal value of $4e^2/\pi h$. We note that in previous works, this universal value is already obtained at the Drude level. The topological disorder discussed in this work sets as an initial condition of the RG flow a rather different—disorder dependent—value that can—or not—flow to the usual fixed point.[70] The analysis of the quantum corrections to the conductivity [Eq. (46)] is beyond the scope of this work and will be worked out in the future.

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APPENDIX: SIGMA MODEL CALCULATIONS

Once we have calculated the value of the leading configuration of Q_n from the saddle point Eqs. (35), we expand the action [Eq. (32)] around this value, setting $Q_n=\langle Q_n\rangle+\delta Q_n$ and retain in the expansion terms up to second order in δQ_n:

$$S \approx \langle S\rangle + \delta Q_n \frac{\delta^2 S^*}{\delta Q_n \delta Q_m}\delta Q_m + \cdots . \tag{A1}$$

The finite density of states of Eq. (41) allows us to use the usual identity in the integration of momentum derived for the two-dimensional electron gas:

$$\int \frac{d^2k}{4\pi^2} \to \rho(0)\int d\varepsilon_k, \quad v_F k \to \varepsilon_k .$$

As a consistency test, computing the density of states with this change we get the expression in Eq. (41). This change in variables will simplify the calculations. The * in Eq. (43) means that the functional derivative is evaluated at the saddle point solution for Q_n and Π_n. Also, other terms of quadratic order appear in Eq. (43) with crossing functional derivatives in the fields δQ and $\delta\Pi$. We will see shortly that these derivatives are zero and the former fields are not coupled. These derivatives are

$$\begin{aligned}\delta Q_n \frac{\delta^2 S^*}{\delta Q_n \delta Q_m}\delta Q_m = &-\frac{1}{2}\sum_n \Gamma_n \delta Q_n^2 + \frac{1}{4}\int (dp)\\ &\times (dq)\sum_{n,m}\Gamma_n\Gamma_m \chi_p(k)\chi_m(p\\ &+q)\delta Q_n \,\mathrm{Tr}\, G(k)G(p+q)\delta Q_m\end{aligned} \tag{A2}$$

and

$$\begin{aligned}\delta \Pi_n \frac{\delta^2 S^*}{\delta \Pi_n \delta \Pi_m}\delta \Pi_m = &-\frac{1}{2}\sum_n \Gamma_n \delta \Pi_n^2 - \frac{1}{4}\int (dp)\\ &\times (dq)\sum_{n,m}\Gamma_n\Gamma_m \chi_n(p)\chi_m(p\\ &+q)\delta \Pi_n \,\mathrm{Tr}\, \gamma_5 G(p)\gamma_5 G(p+q)\delta \Pi_m .\end{aligned} \tag{A3}$$

Since the spectral functions are peaked at the Fermi energy, we can effectively restrict the **q** integration to values around the Fermi point, $q \ll K_F$ and write $\chi_m(p+q)\approx\chi_m(p)$ with an error of the order of $O(q)$. In Eq. (A2) the integral will be dominated by the product $G^R G^A$, i.e., by the off diagonal sector of the fields δQ^{+-} leading to the diffusive pole behavior for the fields Q_n. By contrast, in Eq. (A3) using the symmetry property $\gamma_5 G^R \gamma_5 = -G^A$ we can see that the diffusive pole will come from the product $G^A G^A$ and its retarded-retarded counterpart $\delta\Pi^{++,--}$ will be the channel for the diffusive behavior [also, the minus sign appearing in this symmetry corrects the relative sign between the second terms in Eqs. (A2) and (A3)].

Let us define the quantity $C_{mn}(\eta,q)$ as

$$C_{mn}(\eta,q) = \frac{1}{4}\int (dp)\chi_n(p)\chi_m(p)\,\mathrm{Tr}\, G^R(p)G^A(p+q). \tag{A4}$$

To obtain an action for the modes Q_n and Π_n at small q and η, we expand G^R up to first order in η and to second order in q in Eq. (A4).

The first term in this expansion, without any reference to the replica index, is

$$C^0_{nm} = \frac{1}{4}\int (dp)e^{i(n+m)\theta}\,\mathrm{Tr}\begin{pmatrix}\frac{1}{v_F p - i\epsilon} & 0\\ 0 & \frac{1}{-v_F p - i\epsilon}\end{pmatrix} \times \begin{pmatrix}\frac{1}{v_F p + i\epsilon} & 0\\ 0 & \frac{1}{-v_F p + i\epsilon}\end{pmatrix}.$$

We have denoted $\epsilon=\eta+\frac{1}{2\tau}$. In terms of these variables, we have

$$C^0_{nm} = \frac{1}{4}\rho(0)\delta_{-nm}\frac{1}{\eta+\frac{1}{2\tau}}. \tag{A5}$$

Expanding Eq. (A5) up to first order in η, and using (41), we get

$$C^0_{nm} = \frac{\delta_{-nm}}{4gv_F^2\Gamma_0} - \frac{1}{4}\rho(0)\delta_{-nm}(2\tau)^2\eta + O(\eta^2). \tag{A6}$$

The constant term in Eq. (A6) coincides with the term proportional to δQ_n^2 and $\delta\Pi_n^2$ in Eqs. (A2) and (A3), respectively. This mass contribution to the action is

$$\mathcal{L}_m \equiv \frac{1}{4gv_F^2}\sum_n \Gamma_n\left(\frac{\Gamma_n}{\Gamma_0}-1\right)(\delta Q_n^2 + \Pi_n^2). \tag{A7}$$

We immediately see that only the modes δQ_0 and $\delta\Pi_0$ are massless, and they will responsible for the diffusive behavior of the system exactly as happens in the 2DEG.[68] In what follows, we will eliminate the Π field. It represents another diffusion channel that does not mix with the Q's and plays the same role. We will simply multiply by two the final result. The next term in the q expansion reads

$$\begin{aligned}C^1_{nm} = \frac{1}{4}\int (dp)e^{i(m+n)\theta}v_F q\cos\theta\Bigg(&\frac{-1}{(v_F p - i\epsilon)(v_F p + i\epsilon)^2}\\ &+ \frac{1}{(-v_F p - i\epsilon)(-v_F p + i\epsilon)^2}\Bigg).\end{aligned} \tag{A8}$$

Note that in the case of short-ranged isotropic scattering this term vanishes due to the angular integration. In our case, however, the presence of $e^{i(m+n)\theta}$ allows linear terms in q, coupling the massive modes $\delta Q_{\pm 1}$ to δQ_0. The integral in Eq. (A8) is performed changing to the energy variable and noticing that the angular integration gives a nonzero result only when $n=-m\pm 1$

$$C^1_{nm} = \frac{1}{4} v_F q(\delta_{nm-1} + \delta_{nm+1})\rho(0) \int d\varepsilon_p \frac{1}{(\varepsilon_p - i\epsilon)(\varepsilon_p + i\epsilon)^2}, \tag{A9}$$

or, after setting $\eta=0$,

$$C^1_{nm} = -\frac{v_F q}{4}\rho(0)(\delta_{nm-1} + \delta_{nm+1})(2\tau)^2. \tag{A10}$$

To compute the next term in the q expansion, we will use the following trick. The trace of the product of Green functions in (A4) can be explicitly written as

$$\mathrm{Tr}\, G^R G^A = \begin{pmatrix} \frac{1}{v_F p - i\epsilon} & 0 \\ 0 & \frac{1}{-v_F p - i\epsilon} \end{pmatrix} \times \begin{pmatrix} \frac{1}{v_F|\mathbf{p}+\mathbf{q}| + i\epsilon} & 0 \\ 0 & \frac{1}{-v_F|\mathbf{p}+\mathbf{q}| + i\epsilon} \end{pmatrix} \tag{A11}$$

or, rearranging signs,

$$\mathrm{Tr}\, G^R G^A = \frac{1}{v_F p - i\epsilon}\frac{1}{v_F|\mathbf{p}+\mathbf{q}| + i\epsilon} + \frac{1}{v_F p + i\epsilon}\frac{1}{v_F|\mathbf{p}+\mathbf{q}| - i\epsilon}. \tag{A12}$$

We immediately see that the second term in the right hand side is the complex conjugate of the first term, thus,

$$\mathrm{Tr}\, G^R G^A = 2\,\mathrm{Re}\left(\frac{1}{v_F p - i\epsilon}\frac{1}{v_F|\mathbf{p}+\mathbf{q}| + i\epsilon}\right). \tag{A13}$$

The terms already calculated in the expansion of $C_{nm}(\eta,q)$ can be easily derived with this trick, but where we make a real profit of this simplification is in the calculation of the term q^2

$$C^2_{nm} = \frac{1}{2}q^2\,\mathrm{Re}\int (dp) e^{i(m+n)\theta}\frac{1}{v_F p - i\epsilon} \times \left(\frac{\cos^2\theta}{(v_F p + i\epsilon)^3} - \frac{\sin^2\theta}{2v_F p(v_F p + i\epsilon)^2}\right). \tag{A14}$$

If we compare the angular part of Eq. (A14) with the corresponding part in Eq. (A8) we see that after performing the integral in Eq. (A14) there are terms of the type $\delta_{-nm\pm2}$ together with terms δ_{-nm} which generate couplings between the zero modes δQ_0 and the massive $\delta Q_{\pm2}$, and $\delta Q_0\delta Q_0$, respectively. The couplings involving $\delta Q_{\pm2}$ being of order q^2 will produce terms of order q^4 and can be neglected. We will only keep the terms independent of θ in Eq. (A14), from which we will extract the diffusion coefficient D for the massless diffusive mode δQ_0. The result for Eq. (A14) only taking into account the terms proportional to δ_{-nm} is (we shift the pole at $v_F p=0$ in the second term in the integrand and take the real part, the first term will not contribute to this real part)

$$C^2_{nm} = \frac{v_F^2 q^2}{8\pi}\delta_{-nm}\rho(0)(2\tau)^3. \tag{A15}$$

Collecting all the terms, the action for the modes δQ_0 and $\delta Q_{\pm1}$ is

$$\begin{aligned}\delta S_Q \approx \int (dq)\frac{1}{4gv_F^2}\sum_{n\pm1}\Gamma_n\left(\frac{\Gamma_n}{\Gamma_0} - 1\right)\delta Q_n^2 \\ - \frac{\rho(0)(2\tau)^2}{4}\Gamma_0^2\delta Q_0\left(\eta + \frac{v_F^2(2\tau)q^2}{2\pi}\right)\delta Q_0 \\ + \frac{\Gamma_1\Gamma_0\rho(0)v_F q(2\tau)^2}{4}(\delta Q_0\delta Q_1 + \delta Q_0\delta Q_{-1}).\end{aligned} \tag{A16}$$

In order to get a theory for the $n=0$ modes, we integrate out the $n=\pm1$ modes in Eq. (A16). Using again Eq. (41) we get

$$\delta S = \int dq \frac{\rho(0)(2\tau)^2\Gamma_0^2}{4}\delta Q_0\left(\eta + \frac{v_F^2 q^2(2\tau)}{2\pi} + \frac{\Gamma_1}{\Gamma_0 - \Gamma_1}\frac{v_F^2 q^2(2\tau)}{2\pi}\right)\delta Q_0. \tag{A17}$$

If we simplify and use again Eq. (41) we arrive to the final action for the massless modes

$$\delta S = \int dq\,\delta Q_0\frac{(2\tau)\Gamma_0}{4gv_F^2}(\eta + Dq^2)\delta Q_0, \tag{A18}$$

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